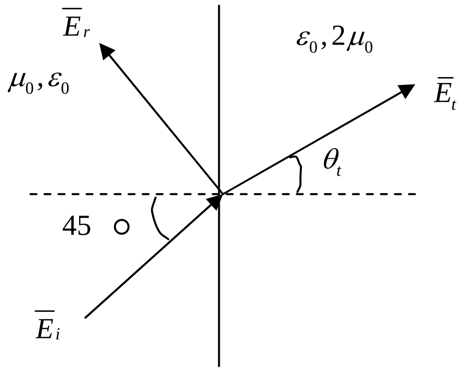
Prob.1

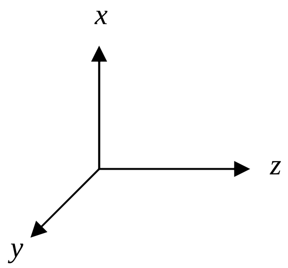
An EM wave is incident on dielectric interface as shown in the figure. There are no surface currents on the interface.

(a)(10) Determine the angle of transmission.

(b)(10) We learned in the class that 1+Γ=τ. Derive another equation that relates Γ and τ from the boundary condition on $\overline{H}_{\tan}$. (Hint : Obtain the tangential component of $\overline{H}$ from $\nabla \times \overline{E} = -j\omega\mu\overline{H}$. What condition should be satisfied in order for this tangential component to be continuous?)

(c)(10) Determines the numerical ratio of reflected power to incident power.





$$\vec{E}_i = \overline{y}e^{-jk_{1,x}x}e^{-jk_{1,z}Z}$$

$$\vec{E}_r = \overline{y}\Gamma e^{-jk_{1,x}x}e^{+jk_{1,z}z}$$

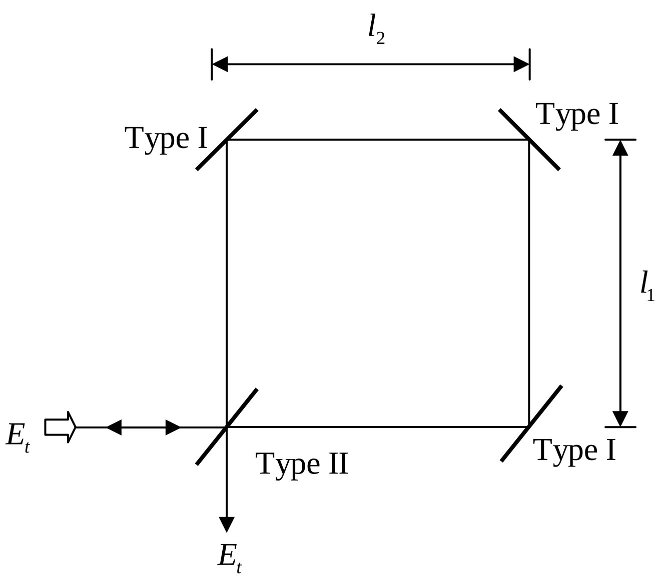
$$\vec{E}_t = \overline{y}\tau e^{-jk_{2,x}x}e^{-jk_{2,z}z}$$

Prob. 2

An interferometer is made up of two types of mirrors as shown below. Type I mirror has $\Gamma_1=-1$ and Type II mirror has $\Gamma_2 = 1/\sqrt{2}$ and $\tau_2 = j\,1/\sqrt{2}$.

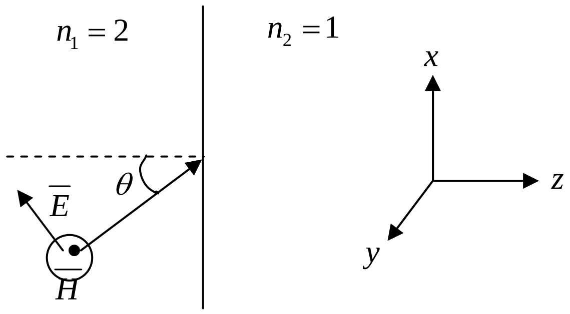
(a) Determine E_t when $l_1=l_2$.

(b) Determine the ratio of the total reflected power to the incidenr power when $l_1=2l_2$.



Prob. 3

A plane EM wave having parallel (TM) polarization is incident on a lossless dielectric interface as shown below.

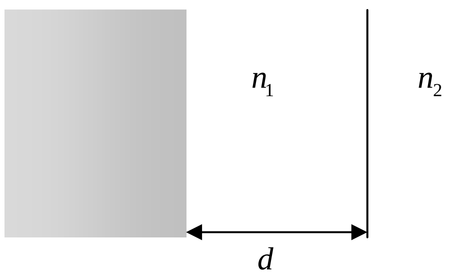


(a)(10) Assume Θ is less than the Brewster angle. Sketch the reflected EM wave. Make sure the directions for E- and H-field are correctly given.

(b)(10) Now the incident angle is 45 deg. and the incident EM wave has wavelength of 1μm in vacuum. What is the z-component of the k-vector in the medium located on the right side? Give a numerical answer with an appropriate unit.

(c)(15) For the conditions given in (b), what is the reflection coefficient Γ? Derive its expression first and then give a numerical answer.

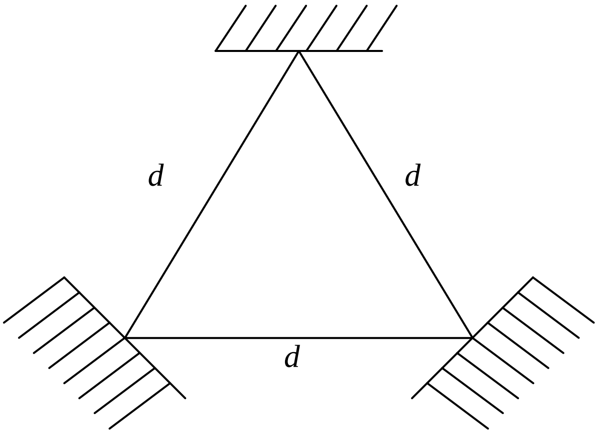
(d)(10) A waveguide is formed by placing a sheet of perfectly conducting metal in parallel with the dielectric interface as shown below. All the conditions are the same as in (b). Derive an expression for k_z of the **fundamental guided mode**. Give your answer in terms of Γ determined in (c) and other symbols given in this problem as well as constants.



(e)(10) Sketch the photon distribution for the 2nd lowest mode in the waveguide given in (d).

Prob. 4

A ring interferometer is made with three perfectly reflecting metallic mirrors located in vacuum as shown below.



(a)(10) If light is somehow injected into the interferometer, resonance is possible only for light with certain frequencies. Determine the resonance condition on the frequency, f. Give your answer in terms of c, the speed of light in vacuum, d, the mirror separation and other constants.

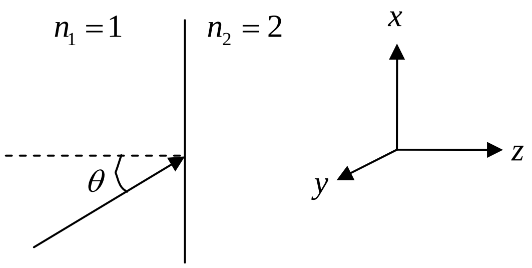
(b)(10) Determine the frequency separation between adjacent resonance modes, Δf.

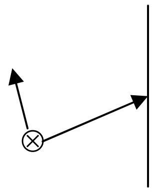
(c)(10) The vacuum is filled with a certain gas whose reflective index has dependence on lightwave frequencies around ω_0: $n(\omega) = n_0 + n' (\omega - \omega_0)$. What is the group velocity for light with frequency ω_0 in this gas? Give your answer in terms of c, the speed of light in vacuum, and symbols given in this problem and other constants.

(d)(15) What is the mode separation for the adjacent resonance modes, Δf, for the conditions given in (c)? (Hint: Use the result obtained in (c).)

Prob. 5

A plane wave whose electric field is given as $\underline{E} = (\underline{x} + \underline{y} - \underline{z}) \exp(-jk_x x - jk_z z)$ is incident on a dielectric interface as shown below.





Example for b,c

(a)(10) Show that the incident angle should be 45 degrees.

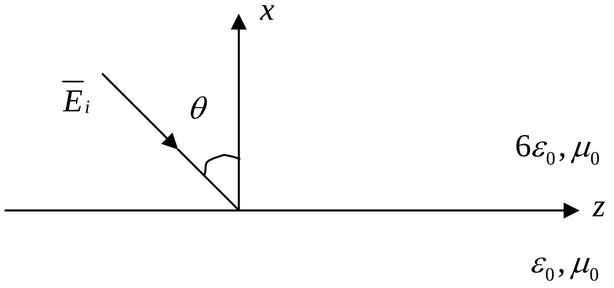
The incident E–field can be decomposed into perpendicular and parallel components.

(b)(10) Sketch directions of reflected E– and H–fields for the perpendicular component. For this, assume x=z=0 for your sketch. A sample sketch for an incident wave is shown above.

(c)(10) Do the same for the parallel component. Also assume x=z=0.

Prob. 6

A plane wave is incident at an angle Θ on a dielectric boundary as shown below. Its electric field is given as $\underline{E_i} = E_0\,(\underline{x}\sin\Theta + \underline{y} + \underline{z}\cos\Theta)\exp(jk_x x - jk_z z)$.



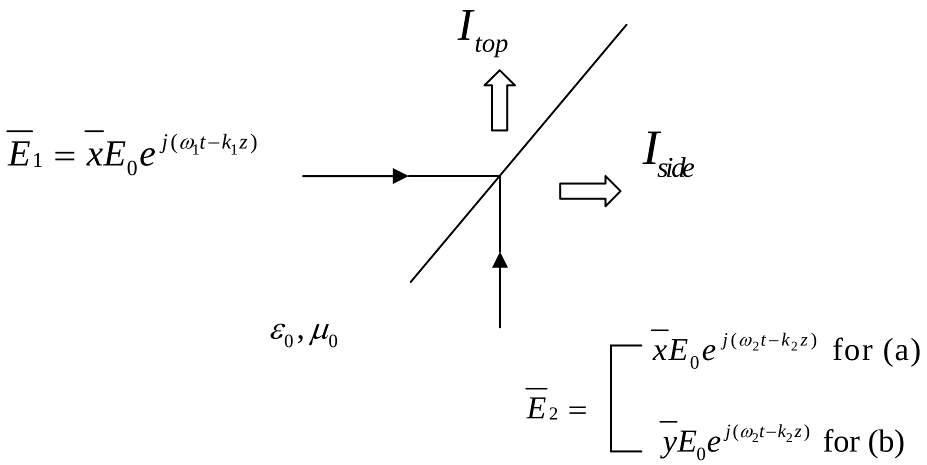
(a)(10) For Θ = 40 deg, what is the polarization of the incident wave; linear, circular, or elliptical?

(b)(10) For Θ = 20 deg., what is the polarization of the reflected wave: linear, circular, or elliptical?

(c)(10) For Θ = 30 deg., what is the polarization of the reflected wave: linear, circular, or elliptical?

Prob. 7

Two beams of light, E_1 and E_2, with angular velocities ω_1 and ω_2 are incident on a beam splitter. The beam splitter splits the incident light into two outputs with an equal power regardless of the incident light polarization. The light intensities are measured at the top and side as shown.



(a)(10) If E_1 and E_2 have the same polarization as shown above, what is the light intensity measured at the top?

(b)(10) If E_1 and E_2 have the orthogonal polarization as shown above, what is the light intensity measured at the top?

(c)(10) For the case given in (a), what is $I_{top} - I_{side}$, where I_{top} is the intensity measured at the top and I_{side} is the intensity measured at the side?

Prob. 8

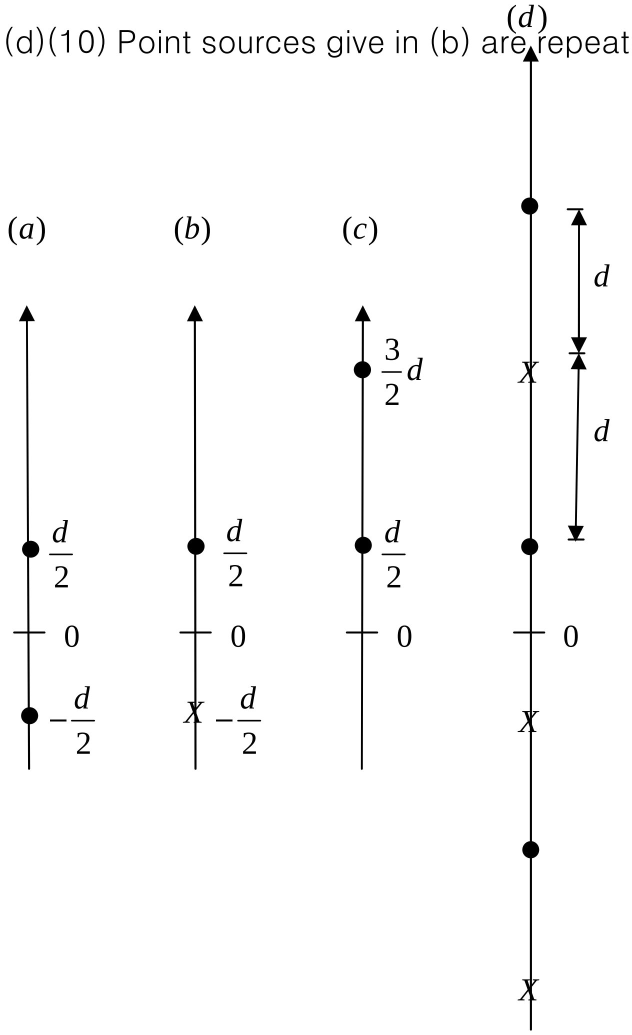
Two point light sources are located as shown below. We are interested in the far-field pattern produced by the interference of these point sources. For each of cases given below, sketch the magnitude of $E(\Theta)/E(0)$. For the sketch, use $\sin(\Theta)/\lambda$ as the x-axis. On the sketch, clearly indicate the locations of the max. and min. magnitudes.

(a)(10) Two source are located near origin and E-fields from two sources are in-phase when they are produced at the source.

(b)(10) Two source are located near origin and E-fields from two sources are out-of-phase when they are produced at the source.

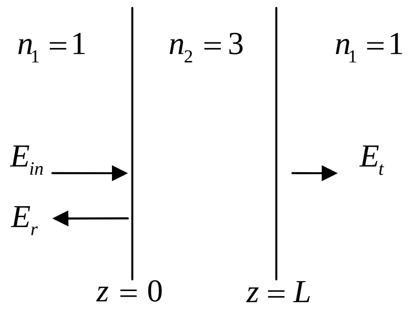
(c)(10) Same as in (a) but the location of sources are shifted by d.

(d)(10) Point sources give in (b) are repeated infinitely.



Prob. 9

A plane wave is incident at a dielectric slab as shown below.



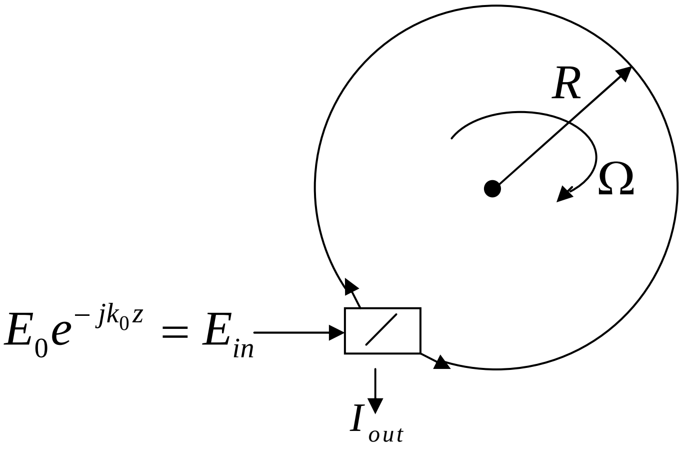
(a)(10) Determine $E_t(z=L)/E_{in}(z=0)$, where E_{in} represents the incident E-field and E_t the total transmitted E-field.

(b)(10) Determine $E_r(z=0)/E_{in}(z=0)$, where E_r represents the total reflected E-field.

(c)(10) Determine $E_t(z=L)/E_r(z=0)$.

Prob. 10

Consider a ring interferometer shown below. The reflected light from the 3-dB beam splitter (R = 0.5) as well as the transmitted light circulates the ring-shaped path and go back into the beam splitter. For simplicity, assume that the total beam path length after the beam splitter is $2\pi R$, and the light travels the ring-shaped path with velocity c.



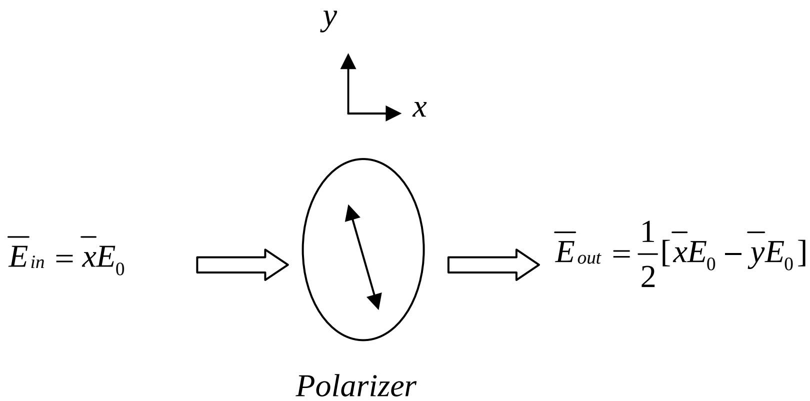
(a)(10) What is the output intensity?

(b)(10) If the ring is rotated with the angular velocity Ω, what is the output intensity?

(c)(10) For what application can this type of interferometer be used?

Prob. 11

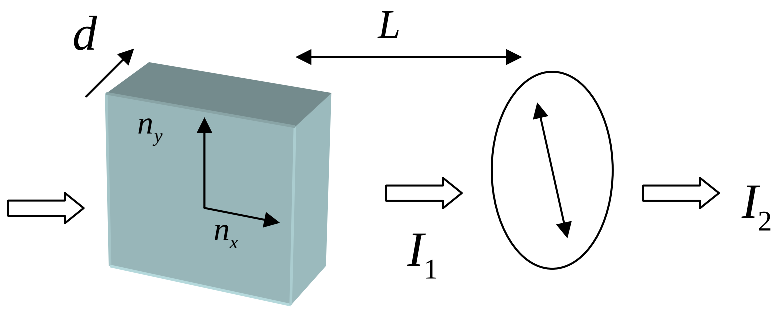
A polarizer is something that allows transmission of an E-field wave polarized only in a certain direction. An example is shown below for a polarizer polarized in x – y direction.



(a)(10) For any input E−field wave, the output of the above polarizer can be expressed as $\underline{E}_{out}$ = J $\underline{E}_{in}$ where J is a 2x2 matrix. Determine J.

An E−field wave is incident on a birefringent material and passes through a polarizer polarized in $\underline{x} - \underline{y}$ direction as shown below.

$$\overline{E}_{in} = (\overline{x} + \overline{y})e^{jk_0 z}$$



$$n_y = n_x + \Delta n$$

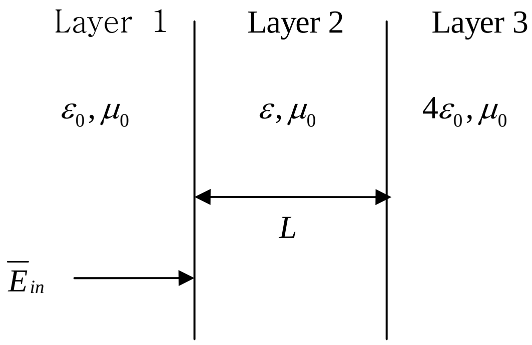
(b)(10) Determine the output intensity, I_1, after the birefringent material but before the polarizer.

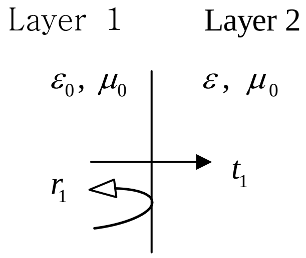
(c)(10) Determine the output intensity, I_2, after passing the polarizer.

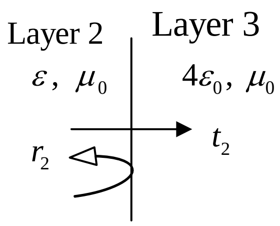
(d)(10) If the birefringence of the material is caused by the E−field applied to the material, for what application can the above set−up be used?

Prob.12

Light with 1μm wavelength is incident on dielectric layers as shown below.







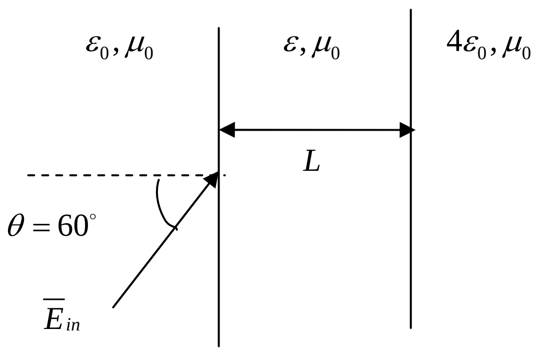
(a) Determine the ratio of transmitted electric field to incident electric field. For your answer, use r_1, t_1 and r_2, t_2 that are reflection and transmission coefficients for Layer 1,2 interfaces and Layer 2,3 interfaces as shown above.

(b) For $\varepsilon=3\varepsilon_0$, determine the smallest possible L for which the reflected power is maximum. (Hint: Make sure you check the signs of r_1 and r_2.)

(c) What is the ratio of reflected power to incident power in the case of (b)?

(d) For a certain combination of L and ε, the reflected power can be zero. Determine numerical values for L and ε. For L, give the smallest possible value.

(e) Now the incident wave is tilted so that the incidence angle is 60 degrees as shown below. Repeat (d) for this case.



Prob. 13

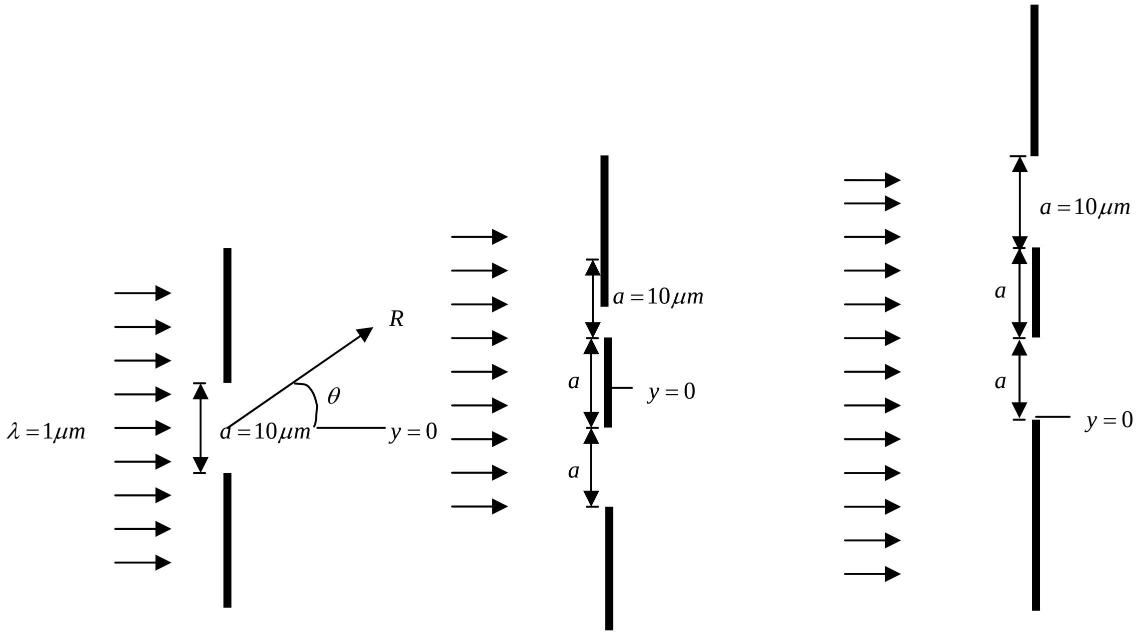
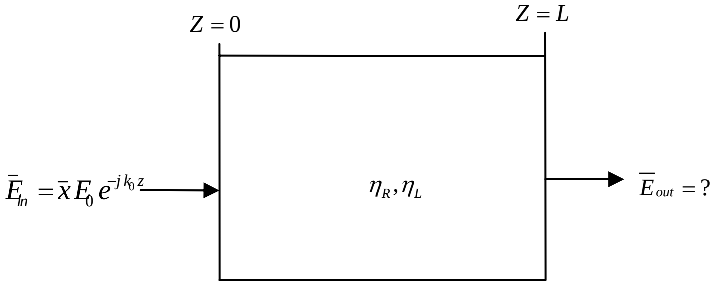


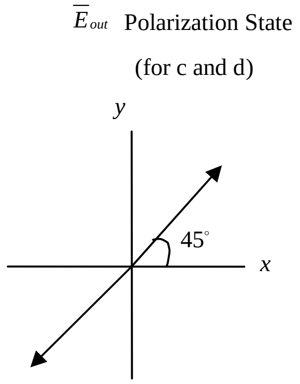
Fig. 2−1 *Fig.* 2−2 *Fig.* 2−3

(a)(10) Sketch the diffracted intensity as a function of sinθ/λ when a plane wave is incident on a slit as shown in Fig. 2-1. At what value (or values) of θ in degrees, the diffracted intensity is zero? Give the θ value having the smallest magnitude.
(b)(10) Determine the minimum (in magnitude) value (or values) of θ in degrees at which the diffracted intensity is zero for the case shown in Fig. 2-2.
(c)(10) Determine θ for which the diffracted intensity is maximum for the case shown in Fig. 2-3.

Prob.14

In certain materials, the value of refractive index depends on the handedness of the incident light polarization. For right-hand circular polarized light, the refractive index of the material is n_R, and for left-hand circular polarized light, it is n_L. Linearly polarized (in x-direction) light is incident on this material as shown below. Assume there is no reflection at the interfaces.





(a)(10) Express the input E-field as a sum of two circularly polarized lights.
(b)(10) The output E-field at z=L can be expressed as
$\underline{E}_{out} = E_{out}(\underline{x}\cos\theta + \underline{y}\sin\theta)$. Determine θ.
(c)(10) Determine the material thickness L so that the output E-field is linearly polarized with 45 deg with x-axis as shown in the above figure. For your answer, give the shortest possible L in terms of λ, n_R, and n_L.
(d)(10) Now, x-direction polarized light is incident at the material at z=L (L is the value determined at (c)) and propagates in –z direction. Sketch the state of polarization at z=0 in the similar manner as shown above.

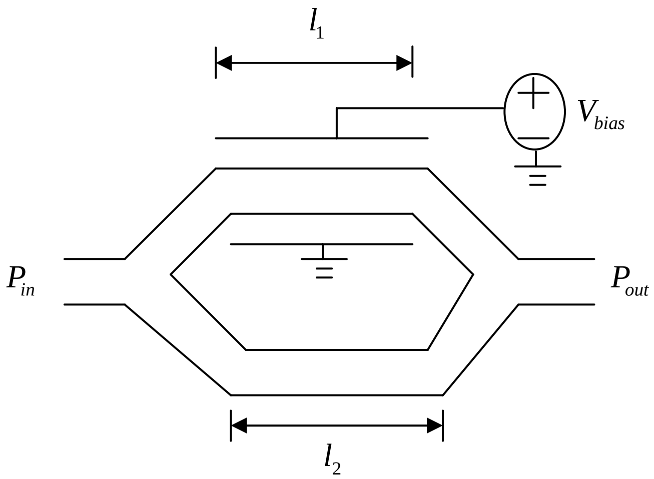
Prob. 15

Consider a Mach-Zehnder interferometer shown below. The refractive index that 1.5 μm light experiences while traveling inside the interferometer is 3.5 when no bias voltage is applied. Due to manufacturing problems, $l_1 = 100\mu m$ and $l_2 = 100.1\mu m$ are not the same.

(a)(10) What is the output power when the input power is $1mW$ at 1.5 μm and no bias is applied?

We want to use the interferometer as an optical on/off switch by applying voltage to the upper arm as shown. The refractive index of the upper arm increases 0.001 per 1 volt applied.

(b)(10) What is the voltage with the smallest absolute value that needs to be applied to make the switch on?



Prob. 16

Consider an electromagnetic wave propagating in the z-direction and described by

$\underline{E}$ = [$\underline{x}$ E_x exp($j\Theta_x$) + $\underline{y}$ E_y exp ($j\Theta_y$)] exp($-jkz$), where E_x, E_y, Θ_x and Θ_y are real numbers.

(a)(5) Let $E_y = j$ and $E_x = \Theta_x = \Theta_y = 0$. What is the polarization of this wave?

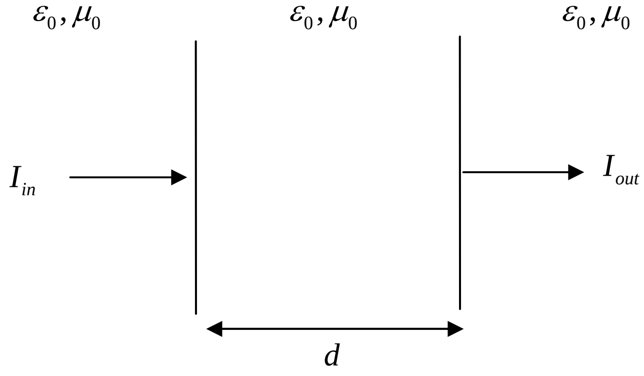
(b)(10) Show that above wave given in (a) can be expressed as a sum of a right-hand and a left-hand circularly polarized wave.

(c)(10) Let $E_x = E_y = 1$ and $\Theta_x = \pi/2$, $\Theta_y = 0$. What is the polarization of this wave? Indicate its handedness, i.e. left- or right-hand.

(d)(5) Show that the above wave given in (b) can be expressed as a sum of two linearly polarized waves.

Prob. 17

Consider a Fabry–Perot interferometer made of two identical partially reflecting/transmitting mirrors as shown below. The mirrors transmit and reflect half of the incident power.



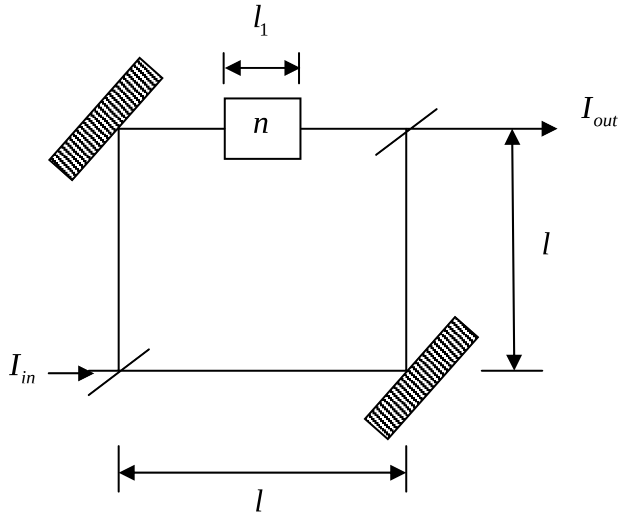
(a)(10) What is I_{out}/I_{in}? Give your answer as a function of sin(kd), where k is the wavenumber in the vacuum and d is the distance between two mirrors.

(b)(10) If the output power is plotted as a function of the frequency of the input light, what is the frequency separation between two adjacent peaks? Express your answer interms of c, speed of light, d, and other fundamental parameters if required.

(c)(10) What is the finesse of this interferometer? Give a numerical answer.

Prob. 18

Consider an interferometer made of two beam spliters and two mirrors. A block of meterial of index n and length l_1 is placed in one arm as shown below. Assume that there is no reflection from this material.

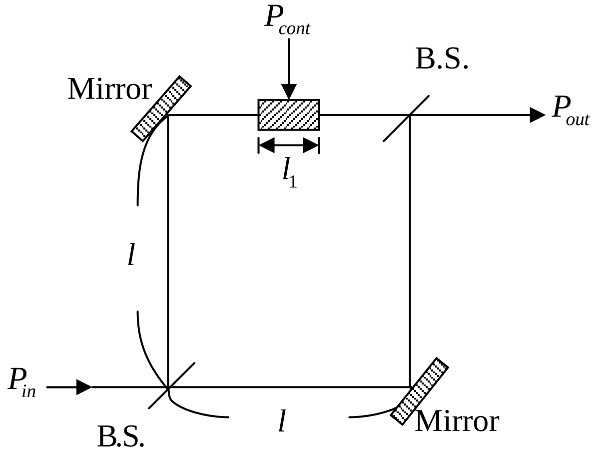


(a)(10) What is I_{out}/I_{in}?

(b)(10) The reflective index of the material linearly changes with the applied voltage, i.e. n(V) = a + bV. We want to determine coefficient b by using the above interferometer. As we apply a voltage to the material, the interferometer output power changes. Measurement shows that 10 V is required to cause the output to shift from the maximum to the adjacent minimum value. What is b? Give a numerical answer. Assume λ= 1μm, l_1 = 1mm, l = 1cm.

Prob. 19

Consider an interferometer made up with two 50% beam splitters and two mirrors. In one arm of the interferometer, a piece of material is inserted whose refractive index can be changed by the external control light as shown below. We want to design an all-optical switch in which P_{out} is turned on and off by P_{cont}. Assume that there is no reflection from the facets of the inserted material.



λ = 1μm

l = 1 cm

l_1 = 5 μm

For , $n = n_1 + n_2 P_{cont}$

$n_1 = 1.05$

$n_1 = 0.1/\text{mW}$

(a)(10) Without any control light P_{cont}, what is P_{out}/P_{in}?

(b)(10) What is the minimum P_{cont} that can produce minimum P_{out}?

(c)(10) What is the minimum P_{cont} that can produce maximum P_{out}?